Stochastic Dependence in Indian Capital Markets: A Fractal Analysis of the CNX Information Technology Index

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INTRODUCTION

This paper employs Lo's (1991) modified rescaled-range analysis and five alternative methods for estimating Hurst exponent (1951), fractal dimension, and Mandelbrot-Lévy characteristic exponent (Lévy 1925) to examine long memory in the CNX Indian technology index. Mandelbrot-Lévy distributions are also referred to as stable, Lévy-stable, L-stable, stable-Paretian, and Pareto-Lévy. Samuelson (1982) popularized the term Mandelbrot-Lévy, but Mandelbrot avoids this expression and the other terms remain current. A new characteristic exponent test for the extremely leptokurtic Cauchy distribution (Mulligan 2000b) is also applied to examine potential Cauchy character in this index. Fractal structure or long memory in equity prices indicates traditional statistical and econometric methods are inadequate for analyzing security markets. Findings have implications for the efficient market hypothesis (EMH), and for the multi-fractal model of asset returns (MMAR) of Mandelbrot, Fisher, and Calvet (1997).

The booming technology sector of the 1990s provides an interesting subject for analysis. It was touted as the "New Economy" (Kelly 1998) not subject to standard economic laws. The technology sector also presented a productivity paradox (Berndt and Malone 1995; Brynjolfsen 1993) in which increased use of computers and other advanced equipment, supposedly motivated by the improved productivity the new technology would provide, never resulted in any measurable productivity gains. An essential and highly variable feature of the New Economy was receiver competence (Eliasson 1985, pp. 47 ff., 57 ff., 1990) or absorbtive capacity (Cohen and Levinthal 1990), referring to the need to make intelligent use of the new technology to realize gains in productivity. Preexisting business strategies formulated for static environments proved inadequate in more dynamic environments (Carpenter and Westphal 2001). The technology sector's dominance by rapid change, technological innovation, and entrepreneurial experimentation may explain the finding of antipersistence in returns for the CNX index. Eliasson (1996) identifies the merger of computing and telecommunications technologies into the internet as the fifth generation of computers. At the time the internet was emerging, many informed actors predicted the fifth generation would be heralded by the introduction of extremely fast and powerful supercomputers. In Japan, for example, investment policy was targeted by the Ministry of International Trade and Industry according to this assumption (Johansson 2001, p. 47). The fact that expectations of informed individuals were so dramatically frustrated by subsequent developments illustrates the dynamic nature of the technology sector, a truly experimentally-organized economy (Eliasson 2001a).

Using the modified rescaled-range (R/S), which is robust against short-term dependence, Lo (1991) found no long memory in stock prices. Technology stocks are of special interest, because they might be less likely to exhibit long memory than other, less volatile, securities. Nevertheless, the high volatility of this equity class makes it an attractive subject for fractal analysis. In applying his modified R/S analysis to equity prices, Lo overturned earlier results based on classical R/S methods finding long memory, but he did not examine the highly volatile technology sector. Mandelbrot (1963a, 1963b) demonstrated all speculative prices can be categorized in accordance with their Hurst exponent H, also called the self-affinity index or scaling exponent (Mandelbrot et al 1997). The Hurst exponent was introduced in the hydrological study of the Nile valley and is the reciprocal of the characteristic exponent alpha (Hurst 1951). Some security prices are persistent with ($0.50 < H < 1.00$). These less-noisy series exhibit clearer trends and more persistence (the closer H is to one), and investors in such assets should earn positive returns. Neely, Weller, and Dittmar (1997) found technical trading rules, formalized with a genetic programming algorithm, provided significant out-of-sample excess returns. However, Hs very close to one indicate high risk of large, abrupt changes, as $H = 1.00$ for the Cauchy distribution, the basis for the characteristic exponent test.

A highly remarkable finding is that the CNX index is antipersistent or mean-reverting with ($0.00 < H < 0.50$), indicating the index is more volatile than a random walk. This indicates the Indian technology sector is promoting

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competition and innovation, and its firms respond to the uncertain environment with experimental and dynamic resource allocation (Eliasson 1991a, 1996, p. 110). If the highly volatile returns are uncorrelated across different asset classes, risk can be minimized by diversification. Antipersistence, that is, H significantly below 0.50, strongly disconfirms the efficient market hypothesis, indicating market participants persistently over-react to new information. Their behavior imposes greater price volatility than would be consistent with market efficiency. Hs significantly above 0.50 would demonstrate stock prices are not random walks, also shedding some doubt on weak market efficiency and indicating technical analysis could provide systematic returns.

Any findings of non-normality or non-Gaussian character would have severe implications for pricing financial derivatives. Because the Black-Scholes (1972, 1973) option pricing model assumes normally-distributed prices for underlying securities, financial derivatives based on non-normal securities prices cannot be priced efficiently with this model. In such a highly volatile environment, advantage accrues to small adaptive firms which can react most quickly in response to market instability or rapid technological change (Piore and Sabel 1984). The validity of agents' information assessments dates rapidly in a highly volatile, non-Gaussian market, even where that information is initially correct. In addition, the finding of antipersistence suggests a more general phenomenon similar to Mussa's (1984) disequilibrium-overshooting model for exchange rate determination.

Long memory series exhibit non-periodic long cycles, or persistent dependence between observations far apart in time. Short-term dependent time series include standard autoregressive moving average and Markov processes, and have the property that observations far apart exhibit little or no statistical dependence. Mandelbrot's R/S or rescaled range analysis distinguishes random from non-random or deterministic series. The rescaled range is the range divided (rescaled) by the standard deviation. Seemingly random time series may be deterministic chaos, fractional Brownian motion (FBM), or a mixture of random and non-random components. Conventional statistical techniques lack power to distinguish unpredictable random components and highly predictable deterministic components (Peters 1999). R/S analysis evolved to address this difficulty by exploiting the structure of dependence in time series irrespective of their marginal distributions, statistically distinguishing non-periodic long-run cyclical dependence from short dependence or Markov character and periodic variation (Mandelbrot 1972a, pp. 259-260). Mandelbrot likens the differences among the three kinds of dependence to the physical distinctions among liquids, gases, and crystals.

Fractal analysis aims at distinguishing deterministic linear behavior from completely unpredictable nonlinear stochastic or probabilistic-chaotic behavior. Somewhere in between lie nonlinear dynamic or chaotic behavior, predictable in the short run but not the long run, and complex processes, predictable in the long run but not the short run (Peters 1999, pp. 164-167). Complex processes exhibit local randomness but global structure, in contrast with nonlinear dynamic processes, which exhibit local regularities but no large-scale structure. Different classes of statistical processes are potentially predictable to different extents, but applying the fractal taxonomy (see Table 1) to correctly categorize the data under consideration is the necessary first step before we can forecast what can be forecast. Respecting the limitations to predictability which inheres in different kinds of statistical behavior is a precondition for constructing meaningful forecasts.

Long memory in equity prices would allow investors to anticipate price movements and earn positive average returns. Fractal analysis offers an alternative to conventional risk measures and permits an evaluation of investment alternatives. Fractal analysis can also identify anti-persistent series, e.g., negative serial correlation. Antipersistent series should also have much shorter cycle lengths than random walks or trend-reinforcing series. Five techniques for estimating the Hurst exponent are reported in this paper, Mandelbrot's (1972a) AR1 rescaled-range or R/S analysis, power spectral-density analysis, roughness-length relationship analysis, variogram analysis, and wavelet analysis. Each method analyzes daily returns on the CNX index as self-affine traces, providing estimates of the Hurst exponent, fractal dimension, and Mandelbrot-Lévy characteristic exponent. The characteristic exponent is then used as a test statistic for the Cauchy distribution. The remainder of the paper is organized as follows. A literature review is provided in the second section. The data are documented in the third section. Methodology and empirical results are presented in the fourth section. Concluding remarks are provided in the fifth section.

LITERATURE

This section describes first, the empirical literature applying fractal analysis to capital markets, then discusses a variety of theoretical expectations of fractal behavior in Indian technology equities over the 1990s, including competence-incompetence mismatching, volatility associated with firm turnover, and high rates of innovation. Finally, the fractal taxonomy of time series, applied below to interpret the empirical results, is developed.

EMPIRICAL APPLICATIONS OF FRACTAL ANALYSIS

The search for long memory in capital markets has been a fixture in the literature applying fractal geometry and chaos theory to economics since Mandelbrot (1963b) shifted his attention from income distribution to speculative prices. Fractal analysis has been applied extensively to equities (Greene and Fielitz 1977; Lo 1991; Barkoulas and Baum 1996; Peters 1996; Koppl et al 1997; Kraemer and Runde 1997; Barkoulas and Travlos 1998; Koppl and Nardone 2001; Mulligan 2004; Mulligan and Lombardo 2004), interest rates (Duan and Jacobs 1996; Barkoulas and Baum 1997a, 1997b), commodities (Cheung and Lai 1993; Barkoulas, Baum, and Oguz 1998), exchange rates (Cheung 1993; Byers and Peel 1996; Koppl and Yeager 1996; Barkoulas and Baum 1997c; Chou and Shih 1997; Andersen and Bollerslev 1997; Koppl and Broussard 1999; Mulligan 2000a), and derivatives (Fang, Lai, and Lai 1994; Barkoulas, Labys, and Onochie 1997; Corazza, Malliaris, and Nardelli 1997).

COMPETENT USE OF MARKET INFORMATION

Ideally, a firm's endeavor should focus on its field of competence. Firms seek to exploit their business environment as competent teams dynamically allocating inputs (Eliasson 1991a), thus production cannot be captured by a static production function (Johansson 2001, p. 15), which necessarily ignores the entrepreneurial element. Coordination performed by firm-level decision makers adds value in each stage of production (Mises 1998 pp. 480-485; Rothbard 1970, pp. 323-332; Garrison 1985, p. 169, 2001, p. 46). The firm's actions are experimental, responding to the uncertain business environment (Eliasson 1996, p. 110). Just as a static environment leads to the implementation of established strategies in the hypothetical construct of the evenly-rotating economy, the more dynamic environment of the Indian technology sector encouraged the development and experimental application of new strategies (Carpenter and Westphal 2001). Firms face environmental uncertainty both, because knowledge and information are always finite resources, and also because this finite resource is asymmetric -- no individual or firm can make use of all available information. Individuals necessarily filter out most of the information they encounter in order to make intelligent and effective use of a limited subset, constructing what Eliasson (1990) calls 'a competence bloc'. Project-oriented management, which facilitates the compartmentalized use of limited information, has long been the paradigm in information technology. Piore and Sabel (1984) suggest market instability promotes competitiveness, and provides an advantage to small firms which can react more quickly in response to market volatility, high uncertainty, or rapid technological change. Highly competent, highly innovative firms should contribute antipersistence to market data processes. Their actions should be expected to impose higher volatility on capital markets because they are engines of Schumpeterian creative destruction (Schumpeter 1934).

In the Indian technology sector, firms compete for information and technical knowledge and then allocate these resources experimentally, competing for the best outcomes. Competent resource allocation is not a conventional optimization process, but a search activity which aims at uncovering an unrealizable optimum. Using Eliasson's (1996) terminology, entrepreneurial managers seek to allocate resources found in the state space to the business opportunity set of profitable outcomes. Entrepreneurs compete to reach the best optima within the partially unexplored business opportunity set, and the more alert entrepreneurs also compete to uncover more of the latent opportunities hidden there. Entrepreneurial incompetence can result in capital (in this case both financial and physical capital) being misallocated, that is, allocated towards unprofitable uses outside the business opportunity set. Furthermore, the very activity of invention, innovation, learning, facilitating customer competence, facilitating competence of venture capitalists, etc., transforms the business opportunity set and continuously makes better optima possible. "Both the state space and the business opportunity set are, however, at each point in time bounded (but expanding through exploration)" (Johansson 2001, p. 18).

Johansson (2001) and Eliasson (1983, p. 274, 1991b) suggest a non-convergence property, characterized by instability of market equilibria, should be "expected in an economy where information use and communication activities dominate resource use and where technological change in information technology dominates total productivity change through constant systems reorganization" (Johansson 2001, p. 121). This characterizes the technology sector, particularly in India, and contrasts markedly with sectors characterized by less competent, less entrepreneurial firms. In contrast to the business enterprises emphasized in traditional economic and managerial theory, likely to exhibit persistence in equity returns, new economy firms are more likely to display antipersistence.

INCOMPETENT MONEY

Information costs are one kind of transaction costs, which Coase (1937, pp. 38-46; 1988, p. 7) identifies as the main reason the division of labor is organized in firms. This implies that the transaction costs avoided through organizing production in firms more than offset inefficiencies imposed by the firm's bureaucratic organization. The mere existence of firms presumptively demonstrates it successfully minimizes transaction costs, at least over the long run. Transaction costs are especially critical in the technology sector, where information obsoletes rapidly. Competence blocs can only persist if resource allocation is flexible, ongoing, and competently informed. Even if knowledge is embodied in the labor force as human capital, without augmentation through ongoing training, this human capital depreciates rapidly through the diffusion of invention and innovation. If a firm's core practices remain unchanged, lowered performance outcomes are likely (Schumpeter 1942; Hannan and Freeman 1984; Tushman and Anderson 1986; Levinthal 1994).

Venture capitalists fund the formation of new firms and expansion of existing firms. In so doing they perform the vital function of recognizing and correctly valuing or pricing innovation (Eliasson and Eliasson 1996b; Eliasson 1997; Johansson 2001). Competent firms are alert to disequilibrium prices which signal opportunities for entrepreneurial discovery (Kirzner 1984a, p.146; 1984b, pp. 160-161; 1997) and exploit the information contained in disequilibrium prices to adjust the production structure. Johansson (2001, p. 23) notes that "incompetent money," that is, "capital not bundled with market knowledge, *probably has a negative effect on firms, since the financial capital then confers power and authority to actors who do not understand the business* (or the competence of the entrepreneur)" (emphasis in original). Johansson suggests government as the primary supplier of incompetent money (Carlsson et al 1981; Bergstrom 1998) but during the nineties, it appears private sources supplied the U.S. technology sector with all the incompetent money it could absorb. This incompetent money may have resulted from an expansionary monetary policy.

The process of industrial innovation includes the allocation and combination of competencies for which no one understands the full extent or implications (Johansson 2001, p. 25). In this connection, Eliasson (1994) describes the labor market as a "market for competencies." Furthermore, the information and knowledge acquired by workers becomes a form of capital (Baetjer 1998, 2000), as does the knowledge embodied in software (Baetjer et al 1993). The technology sector probably leads all others in the significance which attaches to competence blocs, the harm that can be created by competence misalignments, and the difficulty in perfectly juxtaposing adjacent competencies. Thus, misallocation is inevitable, and an essential part of economic progress. It is necessary to contrast naturally unbalanced growth with the misallocation induced by an expansionary monetary policy. Competence possesses the unique property of being self-allocating (Pelikan 1993; Eliasson 1996); incompetence, in contrast, may be described as *self-misallocating*. In a rapidly changing state space, due to technological change or other factors, competence obsoletes rapidly and becomes incompetence if it is not constantly updated. Where allocation is not sufficiently flexible, misallocation must result and must be persistent.

Cheung and Lai (1993) suggest Heiner's (1980) and Kaen and Rosenman's (1986) competence-difficulty (C-D) gap hypothesis as a potential source of long memory in asset prices, offering a theoretical expectation of long memory. The C-D gap is a discrepancy between investors' competence to make optimal decisions and the complexity of exogenous risk, which is widely thought to be especially high for the technology sector. A wide C-D gap leads to investor dependence on deterministic rules, which can lead to persistent price movements in one direction - crashes and speculative bubbles. Due to irregular arrival of new information, Kaen and Rosenman argue persistent price movements may suddenly reverse direction, leading to non-periodic cycles. Persistence in equity returns is thus expected more frequently for larger, more established, less entrepreneurial firms, in contrast to smaller firms with more effectively-delimited competence blocs. Program trading introduces the same phenomenon of persistent returns, and interestingly enough, is more likely to be engaged in for large firms. In addition, many technology investors rely heavily on perceived market sentiment, which is also subject to both persistence and unpredictable reversals.

FIRM SIZE, AGE AND INNOVATION

Researchers have identified the importance of small and medium-sized firms, which typify the Indian technology sector, in driving economic growth (Birch 1981, 1987; Davidsson et al 1994a, 1994b, 1996; Audretsch 1999), as well as documenting negative relationships between firm growth and firm size and/or firm age (Evans 1987a, 1987b; Dunne et al 1987). Kirchhoff (1994) found that these growth effects were strongly amplified for the technology sector. A related line of inquiry has documented decreasing shares of production and employment by large, old, well-established firms, being displaced by increasing shares to large numbers of newer, smaller firms, since about 1970 (Brock and Evans 1986, 1989; Carlsson 1989, 1992; Loveman and Sengenberger 1991; Acs 1996; Acs and Audretsch 1987a, 1987b, 1988, 1990a, 1990b, 1993, 1999; OECD 1996). Thus, we should expect

to observe antipersistent returns for small firms, and persistent returns for larger ones.
Small firms, such as those that dominate the technology sector, act as agents of change (Acs 1992) and tend to be more innovative than larger firms, which often suffer from more bureaucratic organization (Acs and Audretsch 1987a, 1987b, 1988, 1993). Small, innovative firms typically gain "first-mover advantages" (Thomas 1985), though large firms can also be first movers. The level of bureaucratic inertia a firm experiences increases with age and size (Hannan and Freeman 1984). Because firm age and firm size highly correlated, empirical examinations may have difficulty separating these two as causal factors.
Small firms contributed the majority of innovations in the technology sector (Acs and Audretsch 1990a, 1990b), and in some cases the success of these innovations enabled the innovating firm to become a large one, sometimes a less innovative one. Microsoft and Intel typify this evolutionary process in the U.S., Infosys and HCL in the Indian technology sector. Large, established firms, exploiting the comparative advantage that comes from being large and established, generally deepen existing innovations they may have pioneered (Almeida and Kogut 1997; Almeida 1999). This kind of essential, though clearly less innovative activity, should result in more persistent, rather than antipersistent, returns for the larger firms. Though large firms have comparative advantage in extending existing innovations they, eventually diminishing returns must set in. Johansson (2001, p. 71) suggests large firms look for innovative processes, trying to improve what they already do well, whereas small firms look for innovative products, which are more important for long run growth (Acs and Audretsch 1999). Lombardo and Mulligan (2003) note that established firms tend to allocate resources along historical, as opposed to dynamic, patterns.
As a firm grows or ages, it becomes increasingly difficult to alter the competence base of its research functions, supporting the expectation of persistent returns for large firms. Leastadius (2000) suggests large established firms only embrace new technology that complements the organization's existing competence base. New technology which challenges the organization's competence base, or renders it obsolete, will typically be resisted. Because large firms have existing capital structures and knowledge bases to protect, they will be resistant to change which does not complement existing physical and human capital. This distinction is similar to, though more general than, that underlying Bischoff's (1970) "putty-clay" model of investment, which emphasizes the distinction between highly-liquid, uninvested financial capital, such as venture capitalists provide to small, new firms, and highly illiquid, installed capital equipment, such as might be abundant in large established firms. Here, the distinction is generalized to include human capital. Small firms are freer to adapt than large firms because the small firms are not constrained by large illiquid stocks of human or physical capital. Also, an established firm may be more interested in protecting existing economic rents than creating new profits (Geroski 1995, p. 431), also supporting the expectation of persistence. Small new firms will not have rents to protect, and typically during the technology boom in India, new start ups either grew explosively or failed after a short period, contributing an additional source of antipersistence.
Small firms' less bureaucratic organization enables them to better exploit new knowledge and information (Link and Rees 1990; Link and Bozeman 1991). Thus, the technology sector's dominance by small firms leads to a higher rate of innovation, which can be thought of as random exogenous shocks, thus explaining a high level of volatility among technology equities. In reality, however, innovations are neither random nor exogenous, but result from firms' response to their environment, including uncertainty and technological change. Acs et al (1997) suggest small firms contribute more innovation because they better respect and protect the property rights of innovators.

FIRM ENTRY, EXIT AND INNOVATION

Empirical investigations find firm age and size have negative impacts on firm growth rates, and conversely, firm youth and smallness have positive impacts (Davidsson et al 1994a, 1994b, 1996; Liu et al 1999; Heshmati 2001; Johansson 2001). The microeconomic factor of high firm turnover (firm entry combined with firm exit, which frees up resources for better uses) has been found to contribute to macroeconomic growth (Davidsson et al 1994a, 1994b, 1996; Kirchhoff 1994; Reynolds 1994, 1997, 1999; Griliches and Regev 1995; Dunne et al 1987, 1988, 1989; Foster et al 1998; Callejón and Agustí 1999; Callejón and Segarra 1999; Johansson 2001). Audretsch (1995a) concludes that gross firm entry and exit are more important for generating jobs than net firm entry, and Johansson (2001, p. 169) concludes "macroeconomic stability requires microeconomic instability."
With a huge and complex state space, there are always opportunities for realizing large improvements systems productivity through dynamic resource reallocation, most of which occurs through firm entry and exit (Eliasson

[1] The archetypal instance may be IBM's resistance to the PC. They felt they could ride their market advantage in mainframe production indefinitely.

1991a, 1991b; Eliasson and Taymaz 2000; Johansson 2001, p. 119). Allocative improvements are likely to be possible as long as the state space is sufficiently large and complex that market participants, intelligent consumers, skilled workers, inventors, entrepreneurial managers, venture capitalists, cannot marshal so much knowledge and information that they can acquire a dominant comparative advantage over their competitors. It is especially noteworthy that the same situation could arise in a far smaller state space subject to rapid change, such as the innovation-charged technology sector.

A high rate of innovation, even if successfully diffused and adopted, results in rapid resource reallocation and high firm turnover (entry and exit), which may well be observable in equity returns series as greater antipersistence for smaller firms. Rapid changes break down the effectiveness of price signaling in markets, resulting in lost profits through poor or incorrect decisions, and motivating a retreat from decision-making. Eliasson (1990) documents increased search efforts face diminishing returns. When learnable information and knowledge are in rapid flux, there is less incentive for discovery and learning, and firms tend to retreat into established activities, lowering economic growth (Eliasson 1983, 1984, 1991b). Because larger firms should be more successful in implementing this "retreat to habit," they are more likely to exhibit persistent returns. The collapse of U.S. technology equities can thus be seen as a natural process of Schumpeterian creative destruction, rather than a process of correcting the malinvestment triggered by monetary overexpansion, though that phenomenon may also have contributed to a speculative bubble in the technology sector.

Empirical and theoretical studies of firm turnover include Orr (1974), Du Rietz (1975), Baldwin and Gorecki (1989), Acs and Audretsch (1989), and Johansson (2001). Siegfried and Evans (1994) propose the stylized fact that entry increases and exit decreases with firm profitability and growth of local markets. However, Audretsch (1995b) finds firm survival rates lower in highly-innovative markets, such as the Indian technology sector, than in less-innovative markets, though surviving firms have higher growth rates, an outcome also observed by Baldwin (1995).

METHODOLOGICAL APPROACH

Mandelbrot (1972b, 1974) and Mandelbrot, Fisher, and Calvet (1997) have developed the multifractal model of asset returns (MMAR), which shares the long-memory feature of the fractional Brownian motion (FBM) model introduced by Mandelbrot and van Ness (1968). The statistical theory necessary to identify empirical regularities and local scaling properties of MMAR processes with local Hölder exponents is developed by Calvet, Fisher, and Mandelbrot (1997) and applied by Fisher, Calvet, and Mandelbrot (1997). Mandelbrot's (1972a, 1975, 1977) and Mandelbrot and Wallis's (1969) R/S or rescaled range analysis characterizes time series as one of four types: 1.) dependent or autocorrelated series, 2.) persistent, trend-reinforcing series, also called biased random walks, random walks with drift, or fractional Brownian motion, 3.) random walks, or 4.) anti-persistent series.

Table 1 provides the taxonomy of time series identified through fractal analysis. Because the Hurst exponent H is the reciprocal of the Mandelbrot-Lévy characteristic exponent alpha, estimates of H indicate the probability distribution underlying a time series. $H = 1/\alpha = 1/2$ for normally-distributed or Gaussian processes. H = 1 for Cauchy-distributed processes. H = 2 for the Lévy distribution governing tosses of a fair coin. H is also related to the fractal dimension D by the relationship D = 2 - H. In fractal analysis of capital markets, H indicates the relationship between the initial investment R and a constant amount which can be withdrawn, the average return over various samples, providing a steady income without ever totally depleting the portfolio, over all past observations. Note there is no guarantee against future bankruptcy.

Table 1 : Fractal Taxonomy of Time Series

Term	'Color'	Hurst exponent	Fractal dimension	Characteristic exponent
Antipersistent, Negative serial correlation, 1/f noise	Pink noise	$0 \leq H < ½$	$0 < D < 1.50$	$2.00 < \alpha < \infty$
Gaussian process, Normal distribution	White noise	$H \equiv ½$	$D \equiv 1.50$	$\alpha \equiv 2.00$
Brownian motion, Wiener process	Brown noise	$H \equiv ½$	$D \equiv 1.50$	$\alpha \equiv 2.00$
Persistent, Trend-reinforcing, Hurst process	Black noise	$½ < H < 1$	$1.50 < D < 1$	$1 < \alpha < 2.00$
Cauchy process, Cauchy distribution	Cauchy noise	$H \equiv 1$	$D \equiv 1$	$\alpha \equiv 1$

Note: Brown noise or Brownian motion is the cumulative sum of a normally-distributed white-noise process. The changes in, or returns on, a Brownian motion, are white noise. The fractal statistics are the same for Brown and white noise because the brown-noise process should be differenced as part of the estimation process, yielding white noise.

Fractal analysis also gives an estimate of the average non-periodic cycle length, the number of observations after which memory of initial conditions is lost, that is, how long it takes for a single outlier's influence to become immeasurably small. If equity series are random walks with H = 0.50, returns are purely random and should lead to investors' breaking even over the long run. It was found that the series used here, 2795 daily observations over the eleven and one-half years from 1996 to 2007 suggested an average non-periodic cycle length of greater than approximately five years.

DATA

The data are daily closing prices reported by the National Stock Exchange of India (NSE) for the CNX information technology index. The CNX IT index is an actively managed index administered by India Index Services and Products Limited (IISL), a joint venture between NSE and CRISIL. The index is valued according to the market capitalization weighted aggregate method, where the influence of each equity on the index is directly weighted by its market value. The base date of the index is January 1, 1996, and the index is constructed with a base value of 1000. The CNX IT index provides investors and market intermediaries with an appropriate benchmark that captures the performance of the Indian technology sector. The sample period in this study is January 1, 1996 to March 30, 2007 over eleven years of daily data starting with the initial formation of the CNX IT index.

Approximately two cycle lengths of data are necessary for good estimates average non-periodic cycle length using classical R/S techniques (Mandelbrot 1972a; Peters 1994, 1996). Since the average cycle length, if it exists, is not known, this time period offers little potential of including a sufficient number of cycles to allow average cycle length to be definitively measured. With 2,795 daily observations, Lo tests were performed with serial correlation orders up to 1,500. The null hypothesis of no stochastic dependence was rejected for 1 through 15 days of serial correlation, indicating short-term stochastic dependence for up to approximately two weeks following an event, and for 79 through 1,500 days, indicating a longer-term stochastic dependence, memory of which generally persists up to the end of the dataset.

EMPIRICAL RESULTS

This section discusses and interprets the results of the Lo (1991) test for stochastic dependence presented in table 2 and five alternative fractal analysis methods for measuring the Hurst exponent H presented in table 3. The index is first-differenced, losing one observation. Standard errors are given in parentheses. H consistently ranges from zero to 0.50 indicating the CNX index is anti-persistent. Mandelbrot, Fisher, and Calvet (1997) refer to H as the self-affinity index or scaling exponent.

Lo's (1991) modified R/S analysis: Hypothesis tests for stochastic dependence are reported in Table 2. Lo's technique does not provide an estimate for H and rejection of the null hypothesis of no stochastic dependence is necessary to lend any credence to long memory suggested by the five methods for estimating H. Strong evidence is found for stochastic dependence extending approximately two weeks. Interesting, this stochastic dependence disappears after two weeks and *reappears after approximately eighty days* following a shock. The Lo test was carried out to 1500th-order serial correlation, just over half the size of the dataset, and this longer-term memory never seemed to fade. This indicates the average non-periodic cycle length is too long to measure accurately with the sample presently available. The phenomenon of short memory for fifteen days on average, followed by an intermediate memory loss extending from sixteen to seventy-eight days, followed by persistent recall starting on day seventy-nine and extending indefinitely, is particularly interesting.

Table 2 : Lo Test for Stochastic Dependence

AR(n)	Q_n	$P(Q_n)$	*signif*	*AR(n)*	Q_n	$P(Q_n)$	*signif*	*AR(n)*	Q_n	$P(Q_n)$	*signif*
1	1.93729	0.01540	**	61	1.54953	0.14132		121	1.75596	0.04756	**
2	1.81484	0.03355	**	62	1.55220	0.13954		122	1.75858	0.04684	**
3	1.74874	0.04958	**	63	1.55545	0.13739		123	1.76110	0.04616	**
4	1.72855	0.05562	*	64	1.55959	0.13469		124	1.76336	0.04556	**
5	1.73028	0.05508	*	65	1.56385	0.13195		125	1.76552	0.04499	**
6	1.74548	0.05051	*	66	1.56848	0.12902		126	1.76756	0.04445	**
7	1.76582	0.04491	**	67	1.57261	0.12646		127	1.76948	0.04396	**
8	1.78160	0.04094	**	68	1.57629	0.12421		128	1.77088	0.04360	**
9	1.78338	0.04051	**	69	1.57983	0.12207		129	1.77202	0.04331	**
10	1.76902	0.04408	**	70	1.58357	0.11984		130	1.77280	0.04311	**

AR(n)	Q_n	$P(Q_n)$	signif	AR(n)	Q_n	$P(Q_n)$	signif	AR(n)	Q_n	$P(Q_n)$	signif
11	1.74499	0.05065	*	71	1.58743	0.11758		131	1.77350	0.04293	**
12	1.71504	0.06001	*	72	1.59121	0.11539		132	1.77422	0.04275	**
13	1.68294	0.07162	*	73	1.59481	0.11334		133	1.77495	0.04257	**
14	1.65198	0.08452	*	74	1.59858	0.11122		134	1.77566	0.04239	**
15	1.62557	0.09699	*	75	1.60243	0.10910		135	1.77615	0.04227	**
16	1.60513	0.10763		76	1.60678	0.10673		136	1.77681	0.04211	**
17	1.58873	0.11682		77	1.61162	0.10415		137	1.77734	0.04198	**
18	1.57611	0.12431		78	1.61674	0.10147		138	1.77782	0.04186	**
19	1.56930	0.12851		79	1.62241	0.09857	*	139	1.77813	0.04178	**
20	1.56657	0.13023		80	1.62813	0.09572	*	140	1.77823	0.04176	**
21	1.56494	0.13125		81	1.63363	0.09303	*	141	1.77807	0.04180	**
22	1.56393	0.13190		82	1.63872	0.09060	*	142	1.77785	0.04185	**
23	1.56069	0.13397		83	1.64297	0.08862	*	143	1.77763	0.04191	**
24	1.55545	0.13739		84	1.64683	0.08684	*	144	1.77753	0.04193	**
25	1.54878	0.14183		85	1.65092	0.08499	*	145	1.77731	0.04198	**
26	1.54174	0.14664		86	1.65499	0.08319	*	146	1.77731	0.04198	**
27	1.53483	0.15148		87	1.65923	0.08134	*	147	1.77768	0.04189	**
28	1.52983	0.15505		88	1.66341	0.07955	*	148	1.77820	0.04177	**
29	1.52739	0.15682		89	1.66766	0.07776	*	149	1.77868	0.04165	**
30	1.52706	0.15707		90	1.67170	0.07610	*	150	1.77938	0.04147	**
31	1.52747	0.15677		91	1.67557	0.07453	*	151	1.77996	0.04133	**
32	1.52800	0.15638		92	1.67898	0.07317	*	152	1.78064	0.04117	**
33	1.52821	0.15623		93	1.68226	0.07188	*	153	1.78113	0.04105	**
34	1.52814	0.15628		94	1.68576	0.07052	*	154	1.78161	0.04093	**
35	1.52736	0.15685		95	1.68942	0.06913	*	155	1.78230	0.04077	**
36	1.52589	0.15792		96	1.69273	0.06789	*	156	1.78308	0.04058	**
37	1.52462	0.15885		97	1.69580	0.06676	*	157	1.78413	0.04033	**
38	1.52404	0.15928		98	1.69878	0.06568	*	158	1.78519	0.04008	**
39	1.52436	0.15904		99	1.70159	0.06466	*	159	1.78633	0.03980	**
40	1.52506	0.15853		100	1.70443	0.06366	*	160	1.78758	0.03951	**
41	1.52558	0.15815		101	1.70716	0.06270	*	161	1.78896	0.03919	**
42	1.52605	0.15780		102	1.70997	0.06173	*	162	1.79061	0.03881	**
43	1.52709	0.15704		103	1.71241	0.06090	*	163	1.79247	0.03838	**
44	1.52796	0.15641		104	1.71480	0.06009	*	164	1.79433	0.03796	**
45	1.52848	0.15603		105	1.71708	0.05933	*	165	1.79612	0.03755	**
46	1.52885	0.15576		106	1.71921	0.05863	*	166	1.79776	0.03719	**
47	1.52887	0.15575		107	1.72135	0.05793	*	167	1.79951	0.03680	**
48	1.52861	0.15594		108	1.72356	0.05721	*	168	1.80129	0.03640	**
49	1.52835	0.15613		109	1.72590	0.05646	*	169	1.80327	0.03597	**
50	1.52862	0.15593		110	1.72844	0.05566	*	170	1.80521	0.03556	**
51	1.52934	0.15541		111	1.73114	0.05482	*	171	1.80700	0.03518	**
52	1.53121	0.15407		112	1.73403	0.05392	*	172	1.80875	0.03481	**
53	1.53383	0.15219		113	1.73656	0.05315	*	173	1.81057	0.03443	**
54	1.53623	0.15049		114	1.73882	0.05248	*	174	1.81233	0.03406	**
55	1.53884	0.14865		115	1.74133	0.05173	*	175	1.81424	0.03367	**
56	1.54129	0.14695		116	1.74378	0.05101	*	176	1.81607	0.03330	**
57	1.54315	0.14567		117	1.74601	0.05036	*	177	1.81789	0.03293	**
58	1.54485	0.14450		118	1.74831	0.04970	**	178	1.81969	0.03257	**
59	1.54615	0.14361		119	1.75078	0.04900	**	179	1.82140	0.03223	**
60	1.54758	0.14264		120	1.75325	0.04831	**	180	1.82313	0.03190	**
								500	2.20946	0.00213	***
								1000	2.68411	0.00003	***
								1500	2.96362	0.00000	***

The null hypothesis being tested is no stochastic dependence. * indicates rejection at the 10% significance level; ** at the 5%; *** at the 1% level.

Rescaled-range or R/S Analysis: R/S analysis is the traditional technique introduced by Mandelbrot (1972a) to measure the Hurst (1951) exponent H, characteristic exponent α, and fractal dimension D. Time series are classified according to the estimated H, which is defined from the relationship

$$R/S = an^H$$

where R is the average range of all subsamples of size n, S is the average standard deviation for all samples of size n, a is a scaling variable, and n is the size of the subsamples, which is allowed to range from an arbitrarily small value to the largest subsample the data will allow. Putting this expression in logarithms yields

$$\log(R/S) = \log(a) + H \log(n)$$

which is used to estimate H as a regression slope. Results presented in Table 3 provide further difficulty for weak form efficiency all measures indicate H is significantly less than 0.50. Measurable antipersistence demonstrates market participants habitually overreact to new information, and never learn not to. It also suggests the firms included in the CNX index are competent and entrepreneurial, even though many are large, established firms such as Infosys and HCL. These large firms are weighted more heavily in the index due to their high capitalization values.

Normality or Gaussian character is a sufficient condition for weak market efficiency, but not a necessary condition. The result that $H < 0.50$ is generally interpreted as support for the more general multifractal model of asset returns and disconfirmation of the weak-form efficient market hypothesis, which requires $H = 0$. More importantly, findings of $H < 1$ strongly reject weak market efficiency because they demonstrate antipersistence. These findings are absolutely fatal to the Black-Scholes (1972, 1973) option pricing model and its underlying assumption of normally-distributed asset prices. Financial derivatives based on non-normal asset prices cannot be priced efficiently. Thus even if the equity markets for technology stocks are efficient, in spite of substantial empirical evidence against efficiency, the derivatives markets clearly are not efficient. Hs different from 0.50 demonstrate the return series have not been random walks, shedding significant doubt on weak market efficiency and indicating technical analysis could have provided systematic returns. Nevertheless, this finding may be due to short-term dependence still present after taking AR1 residuals, or systematic bias due to information asymmetries, or both.

Table 3 : Estimates of Hurst Exponent H, Characteristic Exponent α, and Fractal Dimension D, Various Methods, for the CNX IT index 1996-2006

	R/S	Power Spectrum	R-L	Variogram	Wavelets
H	0.234	-0.453	0.115	0.018	0.451
Standard Deviation	0.038031	3.278302	0.000801	0.070785	n/a
Alpha	4.274	-2.208	8.696	55.556	2.217
D	1.766	2.453	1.885	1.982	1.549
Z	6.99431	0.29070	480.58919	6.80936	
P(Z)	0.00000	0.38564	0.00000	0.00000	
Significance	***		***	***	

The Z tests are of the null hypothesis of normality, i.e., that $\alpha = 1/2$. Normality is implied by the Efficient Market Hypothesis and the Black-Scholes option pricing model.

Power Spectral Density Analysis: H estimated by this technique is also in the antipersistent range ($H < 0.50$). This method relies on the properties of power spectra of self-affine traces, calculating the power spectrum P(k) where $k = 2p/l$ is the wavenumber, and l is the wavelength, and plotting the logarithm of P(k) versus log(k), after applying a symmetric taper function which transforms the data smoothly to zero at both ends. If the series is self-affine, this plot follows a straight line with a negative slope b, which is estimated by regression and reported as beta, along with its standard error. This coefficient is related to the fractal dimension by: $D = (5 - \text{beta})/2$. H and alpha are computed as $H = 2-D$, and alpha = 1/H. Power spectral density is the most common technique used to obtain the fractal dimension in the literature, although it is also highly problematic due to spectral leakage.

Roughness-Length Relationship: This method is similar to R/S, substituting the root-mean-square (RMS) roughness s(w) and window size w for the standard deviation and range. Then H is computed by regression from a logarithmic form of the relationship $s(w) = w^H$. As noted in Table 3, the roughness-length method provides the most extreme rejection of weak market efficiency. Formal hypothesis tests reject the Gaussian null. One difficulty in applying the roughness-length method is that the standard errors are so low the null hypothesis of $H = 0.50$ is nearly always rejected no matter how nearly normal the asset returns. The seemingly unambiguous rejection of weak market efficiency provided by this technique is best viewed cautiously.

Variogram Analysis: Variogram H indicates antipersistence. The variogram, also known as variance of the increments, or structure function, is defined as the expected value of the squared difference between two y values in a series separated by a distance w. In other words, the sample variogram V(w) of a series y(x) is measured as: $V(w) = [y(x) - y(x+w)]^2$, thus V(w) is the average value of the squared difference between pairs of points at distance w. The distance of separation w is also referred to as the lag. The Hurst exponent is estimated by regression from the relationship $V(w) = w^{2H}$.

Wavelet Analysis: This method was developed by Daubechies (1990), Beylkin (1992), and Coifman et al (1992). Wavelet H estimates indicate antipersistence (H < 0.50). The wavelet method does not provide a standard error for H and cannot be used for hypothesis testing.

Wavelet analysis exploits localized variations in power by decomposing a series into time frequency space to determine both the dominant modes of variability and how those modes vary in time. This method is appropriate for analysis of non-stationary traces such as asset prices, i.e. where the variance does not remain constant with increasing length of the data set. Fractal properties are present where the wavelet power spectrum is a power law function of frequency. The wavelet method is based on the property that wavelet transforms of the self-affine traces also have self-affine properties.

Consider n wavelet transforms each with a different scaling coefficient a_i, where $S_1, S_2, \ldots, S_n$ are the standard deviations from zero of the scaling coefficients a_i. Then define the ratio of the standard deviations $G_1, G_2, \ldots, G_{n-1}$ as: $G_1 = S_1/S_2$, $G_2 = S_2/S_3$, ..., $G_{n-1} = S_{n-1}/S_n$. Then the average value of G_i is estimated as $G_{avg} = (G_i)/(n - 1)$. The estimated Hurst exponent H is computed as a heuristic function of G_{avg}. The Benoit software computes H based on first three dominant wavelet functions, i.e., n is allowed to vary up to 4, and i for the scaling coefficient a_i is allowed to vary from i = 0, 1, 2, 3.

Mandelbrot-Lévy Characteristic Exponent Test: Various statistics are available to test the null hypothesis of normality, but not for the Cauchy distribution, the other extreme. Mulligan (2000b) provides tables of percentages of the Mandelbrot-Lévy characteristic exponent á generated by Monte Carlo experiments with 1,000 iterations for different sample sizes. These critical values can be used to evaluate estimated alphas for the Cauchy null; the null should be rejected if the estimated characteristic exponent lies outside the critical bounds. Dispersion of alpha around the theoretical value of 1.00 varies greatly with the sample size.

The Mandelbrot-Lévy distributions are a family of infinite-variance distributions without explicit analytical expressions, except for special cases. Limiting distributions include the normal, with finite variance, and the Cauchy, with the most extreme platykurtosis or fat tails. Paul Lévy (1925) developed the theory of these distributions. The Hurst exponent H introduced in the hydrological study of the Nile valley is the reciprocal of the characteristic exponent alpha (Hurst 1951). The characteristic function of a Mandelbrot-Lévy random variable is:

$$\log f(t) = i(\delta)t \ \ (\gamma)|t|^{\alpha}[1 + i(\beta)(sign(t)(tan[(\alpha)(\pi/2)])],$$

where δ is the expectation or mean of t if $\alpha > 1$; α is a scale parameter; α is the characteristic exponent; and i is the square root of -1. Gnedenko and Kolmogorov (1954) showed the sum of n independent and identically distributed Mandelbrot-Lévy variables is:

$$n \log f(t) = in(\delta)t \ \ n(\gamma)|t|^{\alpha} [1 + i(\beta)(sign(t)(tan[(\alpha)(\pi/2)])],$$

and thus the distributions exhibit stability under addition. Many applications of the central limit theorem only demonstrate Mandelbrot-Lévy character. The result of normality generally depends on an unjustified assumption of finite variance. Mandelbrot (1972a) introduced a technique for estimating α by regression, further refined by Lo (1991). Mulligan (2000b) estimates the distribution of alpha for Cauchy-distributed random variables. This

Table 4 : Summary Cauchy Distribution Tests
Mandelbrot-Lévy Characteristic Exponent Test

Technique	Estimated alpha	One-tailed critical alphas N = 2500	Outcome
R/S	4.274	α 10% = 1.045 α 5% = 1.066 α 1% = 1.096	Rejects Cauchy H_0 at 1% significance level
Power Spectrum	-2.208	α 10% = 0.899 α 5% = 0.876 α 1% = 0.807	Rejects Cauchy H_0 at 1% significance level

Technique	Estimated alpha	One-tailed critical alphas	Outcome
R-L	8.696	α 10% = 1.045 α 5% = 1.066 α 1% = 1.096	Rejects Cauchy H_0 at 1% significance level
Variogram	55.556	α 10% = 1.045 α 5% = 1.066 α 1% = 1.096	Rejects Cauchy H_0 at 1% significance level
Wavelets	2.217	α 10% = 1.045 α 5% = 1.066 α 1% = 1.096	Rejects Cauchy H_0 at 1% significance level
Note: Critical values from Mulligan (2000b). N_{sample} is 2795. Under the Cauchy null, the distribution of the Mandelbrot-Lévy characteristic exponent alpha varies with the sample size.			

distribution is used to test estimated ás for the CNX IT index against the Cauchy null.

Table 4 shows hypothesis tests on the CNX IT index for the Cauchy distribution. All tests reject the null hypothesis. Strong evidence of Cauchy character for any equity series, particularly an aggregate index, would have been extremely surprising.

CONCLUSION

This paper finds significant evidence of stochastic dependence and antipersistence in the CNX information technology index. This result supports the multifractal model of asset returns (MMAR) and strongly disconfirms the weak form of the efficient market hypothesis. It also suggests that some large technology firms behave in a highly entrepreneurial and innovative manner. Smaller, less-established, more-innovative, more-entrepreneurial firms should exhibit less persistent returns. When equity returns for small, less-established exhibit persistence, the interpretation suggested is that either

(a) information deficits prevent market participants from valuing these equities properly, imposing persistence as traders resort to herding, or

(b) these small firms are not innovative or entrepreneurial, but are mistakenly perceived as such, attracting "incompetent money." If so, these firms, which proliferated during the technology boom, served the useful function of liquidating incompetent money and moving that capital into more competent hands.

Equities traded in efficient markets should have Hurst exponents approximately equal to 0.50, indicating prices change in a purely random, normally-distributed manner. Securities with significant secular trends and non-periodic cycles should display time persistence with $H > 0.50$, unless market efficiency imposes randomness and normality anyway.

Evidence of such prevalent antipersistence tends to disconfirm the efficient market hypothesis and support the more general multifractal model of asset returns (MMAR). Rejection of the null of normality contradicts the efficient market hypothesis in its weak form, and suggests Indian technology equities cannot be efficiently priced. The conclusion suggested is that market participants are incapable of efficiently valuing some technology equities, though not necessarily all. Disconfirmation of the efficient market hypothesis in its weak form suggests possibilities for constructing nonlinear econometric models for improved price forecasting and option valuation.

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