Comparison of VaR Methods : The Case of Indian Equities

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Abstract

Different approaches to calculate VaR are based on different assumptions. This study dealt with a comparative evaluation of four Value-at-Risk models namely, historical VaR, normal VaR, GARCH (1,1) VaR, and volatility weighted historical simulation (VWHS) VaR in terms of their prediction accuracy for an active portfolio of Indian equities. Daily NAVs of 34 Indian equity growth mutual fund schemes for a period of 10 years were used to calculate 95% VaR and backtest the results using Kupiec's POF test for all four VaR models. To identify the better performing VaR methods accurately, the analysis was performed in two phases : pre-crisis analysis and post crisis analysis. We concluded that there was a significant (insignificant) difference in performance of different VaR models if market conditions during VaR calculation and VaR backtesting periods were in contrast (congruence) to each other. The study found VWHS to be a better methodology for measuring VaR of an active portfolio of Indian equity stocks in both phases of the analysis. The results are relevant for traders & retail and institutional investors who hold stocks of Indian companies in their portfolio and need to calculate VaR as a measure of market risk for their positions.

Key words : **backtesting, historical VaR, Kupiec's POF test, GARCH (1,1) VaR, volatility weighted historical simulation VaR, normal VaR, value at risk**

JEL Classification : C52, C53, C14, C15, G32

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Globalization and financial sector reforms in India led to integration of the Indian stock market with the world economy and added new dimensions of risk to stock returns. Enhanced and swift flow of information resulted in a complicated environment where traditional risk measures proved to be insufficient. Two basic considerations of any investment decision are risk and returns. While the calculation of rate of return is quite straightforward, capturing risk is more complex. Risk measurement is an important aspect of financial risk management, as the accuracy of predicted risk measures is crucial for managing the actual future risk exposure. In financial institutions, risk measures are applied for setting internal trading risk limits as well as for calculating the regulatory required capital buffer, consistent with their financial risk. Forecasting accurate risk

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measures is ,therefore, essential because risk overestimation could result in decreased profits as excess retained capital could have generated earnings, whereas risk underestimation could lead to large unexpected financial losses and in extreme cases, regulatory actions and bankruptcy. During the past two decades, 'Value at Risk' (VaR) has become one of the most popular risk measurement techniques in finance. VaR aims to capture the market risk of assets. Market risk is the risk of losses owing to movements in market prices of underlying assets. By market risk, we specifically refer to exposure of the portfolio to losses due to changes in the prices of constituent commodities. These changes may be caused by a variety of exogenous factors like variation in demand and supply of commodities, trade barriers, input costs, tax rates, etc. VaR measures the maximum loss in value for a portfolio over a predetermined time period for a given level of confidence. Selection of an appropriate VaR methodology to capture risk accurately is of prime importance with regards to relevance and utility of VaR as a risk measurement and management tool.

The prime purpose of this study is to examine the accuracy of multiple VaR models for an active portfolio of Indian stocks. The results are relevant in light of the limited literature on evaluation of VaR methods for actively managed equity portfolios in India.

Value At Risk

Since the Basel Committee on Banking Supervision recommended its use as part of the regulatory framework in 1996, Value-at-Risk has become one of the most widely used market risk measures in the financial industry. This success can be related to the fact that VaR can be applied to any financial asset, as it is not dependent on a specific distribution (Daníelsson, 2011). Furthermore, VaR is very intuitively appealing as it can summarize all portfolio risks in a single number, making it easy to both comprehend and communicate to the top management. VaR aims at answering the question: "With a given probability, *p,* what is the expected maximum financial loss on a position over a specific time horizon?"

As per Holton (2003), to specify a value-at-risk metric, we must identify three things:

(i) The period of time over which a possible loss will be calculated - 1 day, 2 weeks, 1 month, etc. This is called the value-at-risk horizon. In our study, the value-at-risk horizon is one trading day.

(ii) A quantile of that possible loss. For example, the portfolio's value-at-risk is usually expressed as a 95% or 99% quantile of loss. In our study, VaR is calculated at 95% quantile, that is, the confidence level is 95%.

(iii) The currency in which the possible loss is denominated is called the base currency.

A loss which exceeds the VaR threshold is termed a "violation," or "exception,", or "Exceedance". For instance, if a portfolio of stocks has a one-day 95% VaR of INR 1000, there is 5% probability that the portfolio will fall in value by more than INR 1000 over a one-day period. This means that a loss of INR 1000 or more on this portfolio is expected on 5 days out of 100 days (because of 5% probability). The confidence level of VaR in this case is 95%.

Back Testing

Despite the wide use and common acceptance of VaR as a risk management tool, the method has been frequently criticized for being incapable to produce reliable risk estimates. When implementing VaR models, there are always numerous simplifications and assumptions involved. Moreover, every VaR model attempts to forecast future asset prices using historical market data which does not necessarily reflect the market environment in the future. Thus, VaR models are useful only if they predict future risks accurately. In order to verify that the results acquired from VaR calculations are consistent and reliable, the models should always be back tested with appropriate statistical methods. Backtesting is a procedure where actual profits and losses are compared to projected VaR estimates. Jorion (2001) referred to these tests aptly as 'reality checks'. If the VaR estimates are not accurate, the models should be reexamined for incorrect assumptions, wrong parameters, or inaccurate modeling.

Literature Review

The specification of return distribution and volatility is central to a VaR model as the accuracy of VaR estimates depends directly on the assumptions made regarding these two crucial inputs in the VaR calculation. The extensive literature on VaR models and backtesting varies broadly on the grounds of these assumptions. Linsmeier and Pearson (1996) provided introduction to the concept of VaR and discussed basic methodologies.

Since the introduction of VaR as a measure of risk, there have been a large number of studies on evaluation and comparison of VaR approaches and models. In fact, there exist numerous methods to calculate VaR and each of these studies considered a definite number of methods for comparison across different international markets and asset classes. A detailed comparison of approaches and methods of calculating VaR was compiled by Abad, Benito, and López (2014). However, the existing literature stands divided about pervasive dominance of any particular VaR approach or method. Studies including those by Danielsson and De Vries (2000) ; Consigli (2002) ; Daníelsson (2002) ; and Bao, Lee, and Saltoglu (2006) produced results in favour of parametric methods, while Giannopoulos and Tunaru (2005) highlighted accurate VaR estimates obtained by the filtered historical simulation. On the other hand, studies such as Gençay and Selçuk (2004) and Nozari, Raei, Jahangiri, and Bahramgiri (2010) showed that the models based on conditional extreme value theory performed better for forecasting VaR.

Some authors show that VaR accuracy depends on volatility models. In the parametric framework, papers like those of González - Rivera, Lee, and Mishra (2004) ; Bali, Mo, and Tang (2008) ; Polanski and Stoja (2009) ; Ergün and Jun (2010) ; and Mancini and Trojani (2011) showed that when more sophisticated volatility models are considered, the VaR estimates are more accurate. Abad and Benito (2013) showed that parametric models can obtain successful VaR measures if conditional variance is estimated properly. Ñíguez (2008) compared the ability to forecast VaR using different generalized autoregressive conditional heteroskedasticity (GARCH) family models and found that the combination of asymmetric models with fractional integrated models provided the best results. Although this evidence is somewhat ambiguous, the asymmetric GARCH models seem to provide better VaR estimations than the symmetric GARCH models.

In the Indian context, many studies have been carried out on evaluation of VaR methods. Sarma, Thomas, and Shah (2003) evaluated performance of a few alternative VaR models, using Nifty as a case study and adopted a bi-direction approach, that is, statistical model selection and model selection based on a loss function. Nath and Reddy (2003) worked on foreign exchange market in India and studied various VaR methods using the Rupee-Dollar exchange rate data to understand which method is best suited for the Indian system. Tripathi and Gupta (2008) tried to find out the accuracy of value at risk model for equity investments in India. The analysis was performed on individual 30 securities of Bombay Stock Exchange (BSE) Sensex and two indices - BSE Sensex and National Stock Exchange's Nifty.

Bhat (2015) compared the performance of alternative models for estimating Value at Risk (VaR) of four different currencies against the Indian rupee and found that VaR models based on an estimate of time-varying volatility performed better than traditional models during turbulent times. Malhotra (2014) estimated the portfolio risks (VaR) using monthly temperature data of three cities namely Delhi, Mumbai, and Chennai. The portfolio risk capital was measured using point backtest method (traffic signal violations approach). The results are relevant for banks which lend to agricultural sector as they can use the findings to manage their capital adequacy requirements for mark-to-market trading portfolios. Das, Basu, and Ghoshal (2009) compared the predictive power of stochastic volatility model (SVM) and Kalman Filter (KF) based approach vis-à-vis exponentially weighted moving average (EWMA) and GARCH based approaches with data from Indian security indices. VaR backtesting results show that SVM significantly outperformed the traditionally recommended EWMA based techniques.

In relation to other studies in comparison to VaR models, we adopted a relatively simpler approach for evaluation of VaR methods. Christoffersen (1998), Christoffersen and Diebold (2000), and Christoffersen and Pelletier (2004) in their work employed conditional coverage tests, whereas, Lopez (1998) introduced loss function. Moreover, we employ basic VaR methods from each type of approach (parametric, semi-parametric, and non-parametric) as compared to other highly sophisticated methodologies. Chou and Wang (2013) suggested an approach that combines quasi-maximum-likelihood fitting of asymmetric conditional autoregressive range (ACARR) models to estimate the current volatility and classical extreme value theory (EVT) to estimate the tail of innovation distribution of the ACARR model. Huang (2015) applied threshold stochastic volatility model to seven major stock indices over a 22-year period and concluded that new model provides reliable estimates and outperforms others.

Ergen (2014) proposed a two-step methodology for value-at-risk prediction and introduced two new models to show that the proposed two-step method outperformed most benchmarks. In contrast to the ongoing development of sophisticated models for VaR estimation, Brooks and Persand (2002) argued that commonly applied methodologies to determine market-based capital risk requirements have certain limitations, and simple methods for calculating VaR often provide better estimates as compared to complex procedures.

Objectives of the Study

The purpose of this paper is to evaluate performance of different approaches and methods of VaR estimation by backtesting, that is, to determine the accuracy of VaR models. We aim to find an appropriate VaR model for an actively managed portfolio of stocks in India. The novelty of the paper lies in the fact that it does not perform backtesting on equity indices but portfolios managed by fund managers. This is in line with practical reality whereby banks have to calculate VaR for their actively traded asset portfolios. Hence, we expect to obtain results of practical relevance in this regard.

How can we assess the accuracy and performance of a VaR model? To answer this question, we first need to define what we mean by "accuracy". By accuracy, we mean: How well does the model measure a particular percentile of the entire profit-and-loss distribution? Since we use data of 34 Indian equity (growth) mutual fund schemes to calculate VaR and perform backtesting in two phases spread over a period of 10 years, the results of this study may well be generalized for Indian equity markets.

Another aim of this study is to verify whether the selection of similar or contrasting data periods in terms of market conditions for VaR calculation and VaR backtesting leads to a difference in the performance of VaR models. It shall also be interesting to know if this difference is large enough to change the preferred model for VaR estimation.

The research hypothesis of the study is that VaR models differ in terms of their accuracy of VaR estimation and that this accuracy depends on the extent of congruence in the market conditions (and thus, in the behavior of returns of underlying asset) during VaR calculation and VaR backtesting periods.

Data and Methodology

The data for this study has been taken from Capitaline database for 34 equity growth schemes pertaining to 20 asset management companies (AMCs) operating in India. Daily net asset values (NAVs) for all schemes were collected for a period of 10 years starting from November 28, 2003 till November 27, 2013. These 34 schemes were largely arrived at by setting a minimum limit of 90% availability of data for analysis.

VaR is a forward looking measure of market risk in the sense that it tries to estimate the maximum potential loss that can be incurred given a confidence level and time horizon. This is one of the main reasons of its wide popularity and acceptance. The irony is that VaR is calculated using historical data. This implies that if VaR is calculated in a relatively tranquil period with positive returns on an average, it produces an estimate which is not suitable for a crisis situation having large negative returns and vice-versa. This indicates that VaR is heavily dependent on historical data and assumes that the returns in near future shall be consistent with the returns in the past. Modifications can surely be made to VaR methods to overcome this issue. In fact, methods which dynamically adjust VaR estimates for volatility changes can produce better results by predicting future returns. Keeping this in view, we decided to divide our 10 years of daily NAV data into two parts: pre-crisis evaluation and post-crisis evaluation.

Our data starting in 2003 and ending in 2013 has a phase of severely distressed returns during the period of financial crisis of 2008-2009. The sub-prime mortgage crisis which originated in the U.S. affected the financial markets in India as well. According to reference dates of business cycles issued by National Bureau of Economic Research (NBER, USA), the U.S. business cycle reached its peak in December 2007, and trough in June 2009. To mark the crisis period accordingly, we considered a period of two years starting from January 1, 2008 till December 31, 2009 to represent the phase of financial distress. The trough might have reached in June 2009, but since the economy takes time to recover and equity returns stabilize gradually, we decided to add another 6 months and stretch the period of distress to December 31, 2009 making it a two-calendar year long phase. VaR backtesting has been applied by adopting the intuitive out-of-sample approach. This means that VaR is first calculated over a period of time and then backtesting is applied ahead of this time to test whether VaR estimates are able to stand the test of time by predicting frequency of violations accurately or not. This out-of-sample approach is appropriate to compare VaR methods because it examines the relevance of a VaR estimate as a forward looking risk measure. It helps us to identify which methods are able to produce future-oriented estimates on the basis of historical data. If a method sustains the out-of-sample backtesting approach and produces lesser number of violations than the level of significance, it is an appropriate method of calculating VaR.

Now, for the first part of analysis, that is, pre-crisis evaluation, we calculated VaR over a four-year period, that is, November 27, 2003 to December 31, 2007 and backtested the estimates over a two-year period of financial crisis, that is, January 1, 2008 to December 31, 2009. This implies that we try to find out which VaR methods produce accurate estimates of VaR (on the basis of historical data during period of relatively stable returns) which are able to withstand backtesting during the turbulent phase of financial crisis spanning two calendar years. For the second part of analysis, that is, post-crisis evaluation, we calculated VaR over a two-year period, that is, January 1, 2010 to December 31, 2011 and backtested the estimates over a 23-month period, that is, January 1, 2012 to November 28, 2013. This implies that we tried to find out if there is a difference between quality of VaR estimates produced by different VaR methods if the VaR calculation period and backtesting period are largely similar in terms of economic conditions. After dividing the data into relevant periods for a two-fold analysis, we calculated VaR by using four methods that collectively represent all three approaches of VaR computation: parametric, non - parametric, semi parametric, and backtesting.

(1) Parametric VaR Methods

(i) Normal VaR : VaR is calculated with the assumption that returns follow a normal probability distribution. The historical volatility is calculated as standard deviation of the return series and plugged in the formula for VaR. The one tailed standard normal variate corresponding to the chosen confidence level is multiplied by the volatility to calculate the extent of deviation from the mean to arrive at a VaR figure.

$$VaR(\alpha) = F^{-1}(\alpha) = \mu_t + \sigma_t(z_\alpha)$$

where, μ represents mean, σ represents historical standard deviation, α denotes confidence level, z_α denotes one tailed z score corresponding to the chosen confidence level and t denotes time period considered.

(ii) GARCH (1,1) VaR : The only difference between normal VaR and GARCH (1,1) VaR is the difference in calculation of volatility to be inserted in the VaR calculation. This methodology produces a forward looking estimate of volatility and hence can predict VaR more accurately if there is volatility clustering in the return series. In this method, σ is calculated using GARCH (1,1). GARCH (1,1) is estimated using the following equations :

$$R_t = \mu_t + \varepsilon_t$$

$$\sigma^2_t = \omega + \alpha_1 [\varepsilon^2_{t-1}] + \beta_1 \sigma^2_{t-1}$$

where, R_t represents return in period t, μ_t represents mean return, and ε_t is error term in mean equation; σ_t is volatility, coefficient ω is constant, α_1 represents ARCH term, and β_1 represents GARCH term.

The selection of GARCH (1,1) among the family of GARCH models is somewhat arbitrary to an extent that we did not do any analysis as to which specification of GARCH would perform better for VaR forecasting. However, we believe that GARCH (1,1) is simpler, parsimonious, and decently powerful in volatility estimation as it gives a good approximation of the observed temporal dependencies in daily data as documented by Andersen and Bollerslev (1998) and Hansen and Lunde (2005). Also, Javed, and Mantalos (2013) claimed that the first lag is sufficient to capture the movements of volatility.

(2) Non-Parametric VaR Method

➢ **Historical VaR :** In this model, VaR is estimated without taking making any assumption about underlying return distribution. It is based on the assumption that history repeats itself. It is the simplest method of estimating VaR. The justification for this methodology is that if returns are stationary, then empirical distribution is a consistent estimator of the unobserved future distribution function. This method is defined as follows :

Consider a sample of past ω returns. The historical VaR at α level of significance for period $t+1$ is given by :

$$Var_\alpha = -Q_{1-\alpha}(r_t, r_{t-1}, \ldots.., r_{t-\omega+1})$$

where, r_t is return of the asset under consideration at time t, and Q is the relevant quantile function at α level of significance.

(3) Semi-Parametric Methods : The process combines the traditional simulation model with conditional volatility models like GARCH (1,1) which makes it attractive in dealing with volatility dynamics. In this category, the following method is used for this study :

➢ **Volatility-Weighted Historical Simulation (VWHS) :** This method was proposed by Hull and White (1998). It combines the benefits of historical VaR with volatility updating. The basic premise of this approach is to update return information with recent changes in volatility. According to this approach, VaR (α) is the α quantile of the distribution of the volatility adjusted returns, where α is the confidence level. In this methodology, all past returns are scaled by volatility adjustment factor. Volatility adjusted returns are calculated as follows :

$$R_{t,i} = r_{t,i} * \frac{\sigma_{T,i}}{\sigma_{t,i}}$$

where, $r_{t,i}$ is actual return for asset i on day t ; $\sigma_{T,i}$ is current forecast of volatility for asset i ; $\sigma_{t,i}$ is volatility forecast for asset i on day t (made at the end of day t-1).

(4) Backtesting of VaR Models : A variety of different testing methods have been proposed for backtesting. Campbell (2007) reviewed a variety of backtests that examine the adequacy of VaR measures. Basic tests such as Kupiec's Proportion of Failure (POF) test (Kupiec, 1995) examined the proportion of losses in excess of VaR. The idea is to count the number of VaR exceptions, that is, days (or holding periods of other length) when portfolio losses exceed VaR estimates. If the number of exceptions is less than what the selected confidence level would indicate, the system overestimates risk. On the contrary, too many exceptions signal underestimation of risk. Naturally, it is rarely the case that we observe the exact number of exceptions as suggested by the significance level. Therefore, it comes down to statistical analysis to study whether the number of exceptions is reasonable or not, that is, will the model be accepted or rejected. This is the technique of backtesting that is used in our study to compare VaR models.

Denoting the number of exceptions as x and the total number of observations as T, we may define the failure rate as x/T. In an ideal situation, this rate would be equal to the level of significance. For instance, if a confidence level of 99 % is used, we have a null hypothesis that the probability of tail losses is equal to $p = (1 - c) = 1 - .99 = 1\%$. Assuming that the model is accurate, the observed failure rate x/T should act as an unbiased measure of p, and thus converge to 1% as sample size is increased (Jorion, 2001).

Each trading outcome either produces a VaR violation exception or not. This sequence of 'successes and failures' is commonly known as Bernoulli trial. The number of exceptions x follows a binomial probability distribution :

$$f(x) = \frac{n!}{x!(n - x!)} p^x (1 - p)^{n-x}$$

where, n is number of trials ; x is number of exceptions ; p is probability of successes.

As the number of observations increase, the binomial distribution can be approximated with a normal distribution:

$$z = \frac{x - pn}{\sqrt{p(1-p)n}} \approx N(0,1)$$

where, pn is the expected number of exceptions and $p(1-p)n$ is the variance of exceptions (Jorion, 2001).

By utilizing this binomial distribution, we can examine the accuracy of the VaR model. However, when conducting a statistical backtest that either accepts or rejects a null hypothesis (of the model being 'good'), there is a trade-off between two types of errors. Type 1 error refers to the possibility of rejecting a correct model and type 2 error refers to the possibility of not rejecting an incorrect model. A statistically powerful test would efficiently minimize both of these probabilities (Jorion, 2001).

Hence, the only information required to implement a POF-test is the number of observations *(n)*, number of exceptions *(x)* and the confidence level *(c)* (Dowd, 2006).

Analysis and Interpretation

We have divided our analysis into two separate heads, that is, pre crisis evaluation and post crisis evaluation. We start by studying the results of pre crisis evaluation. The key phenomenon to keep in consideration is that for pre crisis evaluation, we calculated VaR over a four-year period, that is, November 27, 2003 to December 31, 2007

Table 1. Pre Crisis Evaluation : Summary of Backtesting Results

Category of VaR Method	Non Parametric		Parametric	Semi Parametric
VaR Method	Historical VaR	Normal VaR	GARCH (1,1) VaR	VWHS VaR
No. of Hypotheses 'Not Rejected' out of 34	13	2	1	24
Success Ratio	38.24%	5.88%	2.94%	70.59%

and backtest the estimates over a two year period of financial crisis, that is, January 1, 2008 to December 31, 2009. The Table 1 shows the results of pre-crisis evaluation. The null hypothesis being tested in the Kupiec test is that proportion of failure, that is, x/T (x is the number of violations and T is the total number of observations) is equal to α, that is, level of significance. If this null hypothesis is not rejected, the VaR method passes the Kupiec Test. This is because the total number of violations observed in the testing period is not statistically different from the level of significance. In our study, all hypotheses are tested at 95% level of confidence. For each VaR method, the success ratio has been defined as the ratio of number of schemes for which the null hypotheses is not rejected and total number of schemes, that is, 34.

As can be observed from the Table 1, out of a total of 34 schemes for which VaR has been calculated and backtested, VWHS VaR has been able to accurately predict VaR figures for all 24 schemes, that is, the success ratio of VWHS VaR is 70.59%. This means that for 24 schemes, there is no difference between the proportion of failure observed in the backtesting period and the significance level of 5%. This shows that standardizing the daily log returns by GARCH (1,1) volatility estimates tweaks return series in such a way that it gives accurate VaR estimates calculated as the relevant quantile of the distribution. In relation to other VaR models used in the study, VWHS VaR updates the information present in the returns on a daily basis by using GARCH (1,1) volatility estimates. This process of capturing the latest trends in volatility of returns improves the accuracy of VaR forecasts as reported in the results in Table 1.

The next most accurate model is Historical VaR with a success ratio of 38.24%. This shows that for 13 schemes, for which the null hypotheses have not been rejected, the VaR calculation period has returns low enough to match the distressed returns observed in the backtesting period. The VaR limit was set in such a way that it could not bring a difference in the proportion of violations and level of significance. Parametric VaR models, that is, Normal VaR and GARCH (1,1) VaR have performed poorly in the backtest with a success ratio of 5.88% and 2.94%, respectively. This is mainly due to the use of standard normal variate to determine VaR quantile ; whereas, the return distribution is far from normal.

We now move on to the analysis of results of the post-crisis evaluation. In this case, we calculate VaR for a two year period, that is, January 1, 2010 to December 31, 2011 and backtest the estimates over a 23-month period, that is, January 1, 2012 to November 28, 2013. The period of VaR calculation and VaR backtesting are broadly similar in the sense that none of the periods had experienced a situation of highly distressed or superlative returns. It is interesting to find out whether different VaR methods perform similarly if there is no significant difference between VaR calculation and VaR backtesting period. The Table 2 shows the results of the post-crisis evaluation.

We observe that for post - crisis evaluation, Historical VaR and GARCH (1,1) VaR produce a success ratio of 100%. In fact, normal VaR and VWHS VaR also give highly accurate results with a success ratio of 85.29% and 97.06%, respectively. Broadly, all methods perform well in this scenario where the data used for VaR calculation is quite similar in its behaviour to the data used for VaR backtesting. The superior performance of historical VaR shows that the VaR estimates obtained from the VaR calculation period were accurate as the proportion of violations in the backtesting period were not different from the level of significance. The same is true for GARCH (1,1) VaR and VWHS VaR. However, normal VaR did not perform equally well as the normality assumption for

Table 2. Post Crisis Evaluation : Summary of Backtesting Results

Category of VaR Method	Non Parametric		Parametric	Semi Parametric
Null Hypothesis : *x/T* = Significance Level	Historical VaR	Normal VaR	GARCH (1,1) VaR	VWHS VaR
No. of Hypotheses 'Not Rejected' out of 34	34	29	34	33
Success Ratio	100.00%	85.29%	100.00%	97.06%

returns did not hold true for this analysis either. It is because of this similarity in results that our first analysis of pre-crisis evaluation becomes even more important as it is difficult to differentiate among the relative performance of VaR methodologies in the post-crisis evaluation. This further shows that if the periods for VaR calculation and backtesting are similar in terms of economic conditions, most VaR models may tend to perform adequately well despite differences in their methodology. Hence, with regard to our second objective, we conclude that the extent of congruence in terms of market conditions for VaR calculation and VaR backtesting periods leads to a difference in the performance of VaR models. However, VWHS has emerged as a clear winner in both parts of the analysis. It has accurately predicted VaR estimates in pre-crisis and post-crisis evaluation, delivering sound performance each time.

Hence, we conclude that VWHS is the best methodology out of the four different methods compared in this study for measuring VaR for an active portfolio of Indian stocks as it performs exceedingly well in both parts of the analysis. The study points out the relative precision of semi-parametric methods in comparison to parametric and non-parametric techniques due to incorporation of volatility fluctuations and non-normality of return distribution. It is also equally interesting to note that since VaR models use historical data to produce future risk estimates, their accuracy heavily depends on the congruence between past and future return distributions. Thus, it becomes important to employ VaR models which produce updated risk estimates based on fresh and recent information in returns.

Conclusion and Implications

Value at risk is a widely used measure of risk today because of its simplicity and flexibility. The variants of VaR are evidence of its suitability and universal appeal as a standard measure of risk. It is a measure that needs to be reported by commercial banks as a regulatory compliance according to Basel norms. However, the flexibility of VaR is also one of its limitations. Different approaches to calculate VaR are based on different assumptions. These assumptions relate to the underlying distribution of returns and measurement of volatility. Hence, it is important to identify the most appropriate method of calculating VaR for different assets and markets. In this study, we tried to identify the most suitable VaR methodology for an active portfolio of Indian equities. To get a broad representation of Indian equities, we performed Kupiec's test of backtesting on 34 equity growth mutual fund schemes over a two-fold period separated by the financial crisis of 2008-2009.

The VaR methodologies considered were historical VaR, normal VaR, GARCH (1,1) VaR, and volatility weighted historical simulation (VWHS) VaR. We conclude that there is a noticeable difference in the performance of different VaR models if there is a difference in the market conditions as reflected by highly contrasting behavior of return series in the VaR calculation and VaR backtesting period. On the other hand, if the market conditions and thus, the behavior of returns are similar during the VaR calculation and VaR backtesting period, all VaR methods seem to perform equally well. In our study, VWHS VaR has performed exceedingly well in both scenarios. Hence, we conclude that VWHS VaR is the best methodology out of the four different methods compared in this study for measuring VaR of an active portfolio of Indian equities. The shortcoming of this study is the limited number of VaR methods considered to measure market risk of a portfolio consisting of Indian equities. A major take away from the analysis for research in this area is the importance of accuracy of volatility estimation in order to get better VaR forecasts. Since VWHS VaR is able to estimate volatility more accurately on the basis of historical data out of all the models used in the study, it performs best in VaR estimation. This indicates that any VaR model which accurately captures future volatility in a better manner than VWHS VaR may very well produce better VaR estimates as well. The results are relevant for traders, retail, and institutional investors who hold stocks of Indian companies in their portfolio and need to calculate VaR as a measure of market risk for their positions.

Limitations of the Study and Scope for Further Research

The results give an insight into the relative precision of the three different approaches of VaR modelling for equity portfolios in India. However, the findings should be considered in light of the limitations of the study. Firstly, the study carries out a comparison of only four VaR models out of the umpteen diverse specifications available. There might be much more accurate models than VWHS VaR for VaR calculation for stock portfolios in India. Also, more number of models can be tested simultaneously for each of the three approaches of VaR modelling. Secondly, there are more advanced techniques for backtesting than Kupiec's POF test like conditional coverage tests, loss functions, etc. Each technique has its own advantages, and multiple techniques can be used at the same time. The superiority of any model may not be sustained across all backtesting criteria. Thirdly, the cues for dates to divide the data period into different business cycles were taken from the information given on the website of NBER which corresponds to business cycles in the U.S. The idea behind taking dates as per NBER was the emergence of global financial crisis in the U.S. The methodology of identifying structural breaks according to statistical break point tests was not used to maintain consistency in the analysis of all nine commodities. A different period for crisis may lead to a change in the performance accuracy of VaR models. Lastly, VWHS VaR model has its own share of limitations as it augments the volatility trends in arriving at a VaR figure. A higher volatility forecast would lead to a higher VaR forecast and vice versa. This may lead to over-estimation or under-estimation of VaR if the volatility trend does not sustain in future as suggested by VWHS VaR estimate.

With regard to scope for further research, there are ample possibilities to approach the issue of finding superior models for VaR forecasting. In view of the umpteen models available for VaR estimation, the comparative analysis can be performed using more methods which reflect recent advancements in tail risk estimation. Similarly, improved techniques for backtesting like conditional coverage tests, loss functions, etc., can be applied to find better VaR models. Furthermore, instead of using mutual fund schemes, various investment strategies can be used to form stock portfolios to represent the case of equity asset class in India. The study can also be extended to other asset classes and countries for generalization of results.

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